UNIT-IV

Method I Revised Simplex Method.

① Use Revised simplex method to solve the L.P.P Maximize $Z = 3x_1 + 2x_2 + 5x_3$, Subject to the Constraints : $x_1 + 2x_2 + x_3 \leq 430$; $3x_1 + 2x_3 \leq 460$

$x_1 + 4x_2 \leq 420$, $x_1, x_2, x_3 \geq 0$.

Soln Reduce Std form of L.P.P

$$Z - 3x_1 - 2x_2 - 5x_3 = 0$$

Subj to $x_1 + 2x_2 + x_3 + s_1 = 430$

$3x_1 + 2x_3 + s_2 = 460$

$x_1 + 4x_2 + s_3 = 420$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

Let non basic Variables are x_1, x_2, x_3 and let $x_1 = x_2 = x_3 = 0$

$\therefore s_1 = 430$, $s_2 = 460$, $s_3 = 420$ Basic feasible soln

Coefficient matrices

z-coeff $P_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ x_1-coeff $P_1 = \begin{pmatrix} -3 \\ 1 \\ 3 \\ 1 \end{pmatrix}$ x_2-coeff $P_2 = \begin{pmatrix} -2 \\ 2 \\ 0 \\ 4 \end{pmatrix}$ $P_3 = \begin{pmatrix} -5 \\ 1 \\ 2 \\ 0 \end{pmatrix}$

s_1-coeff $P_4 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ $P_5 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ $P_6 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

non basic Vari x_1 x_2 x_3

Basic Vari s_1 s_2 s_3

Table I

B^{-1}

	Z	S_1	S_2	S_3	Soln	y_3	
Z	1	0	0	0	0	−5	
S_1	0	1	0	0	430	1	$\frac{430}{1}=430$
S_2	0	0	1	0	460	2 (PE)	$\frac{460}{2}=230$ ← PR
S_3	0	0	0	1	420	0	$\frac{420}{0}=\infty$

↓ Pivot column

To Find entering Variable:

= I^{st} row of B^{-1}. (non basic Variable Matrix)

$$= (1\ 0\ 0\ 0) \begin{matrix} x_1 & x_2 & x_3 \\ \begin{pmatrix} -3 & -2 & -5 \\ 1 & 2 & 1 \\ 3 & 0 & 2 \\ 1 & 4 & 0 \end{pmatrix} \end{matrix}$$

1×4 $\quad$ 4×3

$$= \begin{matrix} x_1 & x_2 & x_3 \\ (-3 & -2 & \textcircled{-5}) \end{matrix}$$

Select least negative

Corresponding to x_3 ∴ x_3 entering Variable.

To Find leaves Variable.

Since x_3 entering Variable, determine y_3.

$y_3 = B^{-1}$. (x_3 coeff matrix)

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -5 \\ 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ 1 \\ 2 \\ 0 \end{pmatrix}$$

add this column at last in the above table

least ratio corr to S_2 ∴ S_2 leaves Variable

II nd Iteration

B^{-1}_{new} — non basic x_1, x_2, S_2 ; basic S_1, x_3, S_3

	Z	S_1	x_3	S_3	Soln	y_2	
Z	1	0	$\frac{5}{2}$	0	1150	−2	
S_1	0	1	$-\frac{1}{2}$	0	200	2 (PE)	$\frac{200}{2}=100$ ←
New Pivot row → x_3	0	0	$\frac{1}{2}$	0	230	0	$\frac{230}{0}=\infty$
S_3	0	0	0	1	420	4	$\frac{420}{4}=105$

x_2 entering Var , S_1 leaves Variable

Calculations

Z-coeff
$= 1+5\cdot 0 = 1$
$= 0+5\cdot 0 = 0$
$= 0+5\cdot\frac{1}{2} = \frac{5}{2}$
$= 0+5\cdot 0 = 0$
$= 0+5\cdot 230 = 1150$

S_1-coeffs
$= 0-1(0) = 0$
$= 1-1(0) = 1$
$= 0-1(\frac{1}{2}) = -\frac{1}{2}$
$= 0-1(0) = 0$
$= 430-1\cdot 230 = 200$

S_3-coeffs
$= 0-0$
$= 0$
$= 0$
$= 1$
$= 420$

To Find entering Variable:

$= I^{st}$ row of B^{-1}. (non basic Var Matrix)

$$= \underset{1\times 4}{\begin{pmatrix} 1 & 0 & \frac{5}{2} & 0 \end{pmatrix}} \underset{4\times 3}{\begin{pmatrix} -3 & -2 & 0 \\ 1 & 2 & 0 \\ 3 & 0 & 1 \\ 1 & 4 & 0 \end{pmatrix}} \quad (x_1,\ x_2,\ S_2)$$

$$= \begin{pmatrix} -3+\frac{15}{3} & -2+0+0+0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{6}{3} & -2 & 0 \end{pmatrix} = \begin{pmatrix} 2 & -2 & 0 \end{pmatrix} \quad (x_1,\ x_2,\ S_2)$$

x_2 entering Variable

To Find leaves Variable determine y_2 [since x_2 entering]

$$y_2 = B^{-1}_{new} \cdot (x_2\text{-coeff matrix})$$

$$= \begin{pmatrix} 1 & 0 & \frac{5}{2} & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}_{4\times4} \begin{pmatrix} -2 \\ 2 \\ 0 \\ 4 \end{pmatrix}_{4\times1}$$

$$= \begin{pmatrix} -2+0+0+0 \\ 0+2+0+0 \\ 0+0+0+0 \\ 0+0+0+4 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 0 \\ 4 \end{pmatrix}$$

non basic var x_1 S_1 S_2

basic var x_2 x_3 S_3

III Iteration

B^{-1}_{new}

		Z	x_2	x_3	S_3	Soln	
	Z	1	1	2	0	1350	
New Pivot row →	x_2	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	100	
	x_3	0	0	$\frac{1}{2}$	0	230	
	S_3	0	−2	1	1	20	

New Z-Coeffs

$= 1 + 2\cdot0 = 1$

$= 0 + 2(\frac{1}{2}) = 1$

$= \frac{5}{2} + 2(-\frac{1}{4}) = 2$

$= 0 + 2\cdot0 = 0$

$= 1150 + 2(100) = 1350$

x_3-coeffs

$= 0$

$= 0$

$= \frac{1}{2}$

$= 0$

S_3 coeffs

$= 0 - 4(0) = 0$

$= 0 - 4(\frac{1}{2}) = -2$

$= 0 - 4(-\frac{1}{4}) = 1$

$= 1 - 4\cdot0 = 1$

$= 420 - 4\cdot100 = 20$

now to find entering Variable.

= I^{st} row of B^{-1} (non basic Var matrix)

$$= (1 \quad 1 \quad 2 \quad 0) \begin{pmatrix} x_1 & S_1 & S_2 \\ -3 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$ (Coeff)

$$= (-3+1+6+0 \quad 0+1+0+0 \quad 0+0+2+0)$$

$$= (4 \quad 1 \quad 2)$$

All the values are ≥ 0

∴ Last Iteration table is optimum table.

∴ optimal solution is

$x_1 = 0$, $x_2 = 100$, $x_3 = 230$

Max $Z = 1350$.

---

(2) Use Revised Simplex method to solve the following L.P.P Minimize $Z = x_1 + x_2$ Subject to the Constraints $-x_1 + 2x_2 \geq 7$; $4x_1 + x_2 \geq 6$

$x_1, x_2 \geq 0$.

Soln Given problem is minimization problem Convert in to maximization problem

∴ let $Z^* = -Z$

∴ $Z^* = -x_1 - x_2 - MA_1 - MA_2$

now max Z^* Sub to the given constraints

$x_1 + 2x_2 + A_1 - T_1 = 7$

$4x_1 + x_2 + A_2 - T_2 = 6$, $x_1, x_2, A_1, A_2, T_1, T_2 \geq 0$

number of non basic Var = no. of Vars − no of Constraints

$= 6-2$

$= 4$

Let non basic Variables are x_1, x_2, T_1, T_2

and let $x_1 = x_2 = T_1 = T_2 = 0$

$\therefore A_1 = 7 \quad A_2 = 6$ basic feasible soln

Reduce obj function Z^* purely interms of non basic variables.

$$Z^* = -x_1 - x_2 - M(7 - x_1 - 2x_2 + T_1) - M(6 - 4x_1 - x_2 + T_2)$$

$$Z^* = -x_1 + Mx_1 + 4Mx_1 - x_2 + 2Mx_2 + Mx_2 - MT_1 - MT_2 - 7M - 6M$$

$$Z^* = +(5M-1)x_1 + (3M-1)x_2 - MT_1 - MT_2 - 13M$$

$$Z^* - (5M-1)x_1 - (3M-1)x_2 + MT_1 + MT_2 = -13M$$

$$x_1 + 2x_2 + A_1 - T_1 = 7$$

$$4x_1 + x_2 + A_2 - T_2 = 6$$

Coeffs matrices. (x_1-coeff, x_2-coeff)

Z^* coeff matrix

$$P_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad P_1 = \begin{pmatrix} -(5M-1) \\ 1 \\ 4 \end{pmatrix} \quad P_2 = \begin{pmatrix} -(3M-1) \\ 2 \\ 1 \end{pmatrix}$$

(A_1 coeff, A_2 coeff, T_1 coeff)

$$P_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad P_4 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad P_5 = \begin{pmatrix} +M \\ -1 \\ 0 \end{pmatrix} \quad P_6 = \begin{pmatrix} M \\ 0 \\ -1 \end{pmatrix}$$

non basic Var x_1, x_2, T_1, T_2

basic Var A_1, A_2

B^{-1}

↓ Pivot column

	Z^*	A_1	A_2	Soln	y_1	
Z^*	1	0	0	$-13M$	$-(5M-1)$	
A_1	0	1	0	7	1	$\frac{7}{1}=7$
A_2	0	0	1	6	[4] PE	$\frac{6}{4}=\frac{3}{2}$ least +ve ratio

x_1 entering Var $\therefore$ A_2 leaves Variable

To Find entering Variable

$= I^{st}$ row of B^{-1} (non basic Var matrix)

$x_1 \quad x_2 \quad T_1 \quad T_2$

$$= \underset{1\times3}{(1\ 0\ 0)} \begin{pmatrix} -(5M-1) & -(3M-1) & M & M \\ 1 & 2 & -1 & 0 \\ 4 & 1 & 0 & -1 \end{pmatrix}_{3\times4}$$

$x_1 \quad x_2 \quad T_1 \quad T_2$

$= (\underset{\text{Least -ve}}{\boxed{-(5M-1)}} \quad -(3M-1) \quad M \quad M)$

since M is large +ve

Corresponding to x_1 $\therefore$ x_1 entering Variable.

To Find leaves Vari, determine y_1 $\because$ x_1 entering Variable

$y_1 = B^{-1}$ (x_1 coefficient matrix)

$$y_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -(5M-1) \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} -(5M-1) \\ 1 \\ 4 \end{pmatrix}$$

B^{-1}_{new} non basic Vari A_2 x_2 T_1 T_2

basic Vari A_1 x_1

	z^*	A_1	x_1	Soln	y_2	
z^*	1	0	$\frac{5M-1}{4}$	$\frac{-11M-3}{2}$	$\frac{-7M+3}{4}$	
A_1	0	1	$\frac{-1}{4}$	$\frac{11}{2}$	$\frac{7}{4}$ PE	$\frac{11/2}{7/4} = \frac{22}{7}$ ← least +ve
New Pivot row → x_1	0	0	$\frac{1}{4}$	$\frac{3}{2}$	$\frac{1}{4}$	$\frac{3/2}{1/4} = 6$

Pivot Column

$\therefore x_2$ entering Var A_1 leaves Variable

z-Coeffts

$= 1 + (5M-1)\cdot 0 = 1$

$= 0 + (5M-1)\cdot 0 = 0$

$= 0 + (5M-1)\cdot \frac{1}{4} = \frac{5M-1}{4}$

$= -13M + (5M-1)\cdot \frac{3}{2} = \frac{-26M+15M-3}{2} = \frac{-11M-3}{2}$

Coeff of A_1

$= 0 - 1\cdot 0 = 0$

$= 1 - 1\cdot 0 = 1$

$= 0 - 1\cdot \frac{1}{4} = -\frac{1}{4}$

$= 7 - 1\left(\frac{3}{2}\right) = \frac{11}{2}$

To Find entering Variable

$= I^{st}$ row of B^{-1}_{new} (non basic Vari matrix)

A_2 x_2 T_1 T_2

$$= \begin{pmatrix} 1 & 0 & \frac{5M-1}{4} \end{pmatrix} \begin{pmatrix} 0 & -(3M-1) & M & M \\ 0 & 2 & -1 & 0 \\ 1 & 1 & 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{5M-1}{4} & -(3M-1)+\frac{5M-1}{4} & M & M-\frac{(5M-1)}{4} \end{pmatrix}$$

A_2 x_2 T_1 T_2

$$= \begin{pmatrix} \frac{5M-1}{2} & \frac{-7M+3}{4} & M & \frac{-M+1}{4} \end{pmatrix}$$

Least -ve

$\therefore x_2$ entering Variable.

To Find leaves Variable determine y_2 (since x_2 entering Va

$y_2 = B^{-1}_{new}$ (x_2 coeff matrix)

$$y_2 = \begin{pmatrix} 1 & 0 & \frac{5M-1}{4} \\ 0 & 1 & -\frac{1}{4} \\ 0 & 0 & \frac{1}{4} \end{pmatrix} \begin{pmatrix} -(3M-1) \\ 2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -(3M-1) + \frac{5M-1}{4} \\ 2 - \frac{1}{4} \\ \frac{1}{4} \end{pmatrix} = \begin{pmatrix} \frac{-7M+3}{4} \\ \frac{7}{4} \\ \frac{1}{4} \end{pmatrix}$$

non basic Variable A_2 A_1 T_1 T_2

basic Variable x_2 x_1

		Z^*	x_2	x_1	Soln	
	Z^*	1	$M-\frac{3}{7}$	$M-\frac{1}{7}$	$\frac{-27}{7}$	
New Pivot row →	x_2	0	$\frac{4}{7}$	$\frac{-1}{7}$	$\frac{22}{7}$	
	x_1	0	$\frac{-1}{7}$	$\frac{2}{7}$	$\frac{5}{7}$	

new Z-coeffs

$= 1 + \left(\frac{7M-3}{4}\right) \cdot 0 = 1$

$= 0 + \left(\frac{7M-3}{4}\right) \cdot \frac{4}{7} = M - \frac{3}{7}$

$= \frac{5M-1}{4} + \left(\frac{7M-3}{4}\right)\left(\frac{-1}{7}\right) = \frac{35M-7-7M+3}{28} = \frac{28M-4}{28} = \frac{2(14M-2)}{28\,14} = M - \frac{1}{7}$

$= \frac{-11M-3}{2} + \left(\frac{7M-3}{4_2}\right)\left(\frac{22}{7}\right) = \frac{-77M-21+77M-33}{14} = \frac{-54}{14} = \frac{-27}{7}$

x_1 - Coeff

$= 0 - \frac{1}{4} \cdot 0 = 0$

$= 0 - \frac{1}{4}(4) = -\frac{1}{7}$

$= \frac{1}{4} - \frac{1}{4}\left(-\frac{1}{4}\right) = \frac{7+1}{28} = \frac{8}{28} = \frac{4}{14} = \frac{2}{7}$

$= \frac{3}{2} - \frac{1}{4}\left(\frac{22}{7}\right) = \frac{21-11}{14} = \frac{10}{14} = \frac{5}{7}$

To Find entering Variable.

$= I^{st}$ row of B^{-1}_{new} (non basic vari matrix)

$$= \underset{1\times 3}{\begin{pmatrix} 1 & M-\frac{3}{7} & M-\frac{1}{7} \end{pmatrix}} \underset{3\times 4}{\begin{pmatrix} 0 & 0 & M & M \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}}$$

$$= \left(M-\frac{3}{7} \quad M-\frac{1}{7} \quad M-M+\frac{3}{7} \quad M-M+\frac{1}{7} \right)$$

$$= \left(M-\frac{3}{7} \quad M-\frac{1}{7} \quad \frac{3}{7} \quad \frac{1}{7} \right)$$

All the values are ≥ 0 $\because$ M is large the value

$\therefore$ Last table is optimum table

$\therefore$ optimal solution is

$\therefore$ Maximum $Z^* = -\frac{27}{7}$

$\therefore$ Minimum $Z = -(\text{Max } Z^*)$

$= -\left(-\frac{27}{7}\right)$

Minimum $Z = \frac{27}{7}$

$x_1 = \frac{5}{7}$, $x_2 = \frac{22}{7}$